Looking Beyond Inflationary Cosmology

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Abstract: In spite of the phenomenological successes of the inflationary universe scenario, the current realizations of inflation making use of scalar fields lead to serious conceptual problems which are reviewed in this lecture. String theory may provide an avenue towards addressing these problems. One particular approach to combining string theory and cosmology is String Gas Cosmology. The basic principles of this approach are summarized.

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Résumé : Malgré les succes phénoménologiques du scenario de l'univers inflationnaire, les implémentations actuelles du modèle inflationnaire en utilisant des champs scalaries mènent à des problèmes conceptuels, la discussion desquels forme la première partie de cette conférence. La théorie des supercordes engendre la possibilité de résoudre ces problèmes. "String Gas Cosmology" présente un chemin intéressant pour combiner la théorie des supercordes et la cosmologie. Un sommaire des aspects de base de ce modèle est présenté dans la deuxième partie de cette conférence.

1. Introduction

The inflationary universe scenario is the current paradigm for understanding the evolution of the very early universe. Developed about 25 years ago [1, 2] (see also [3, 4] for earlier but related work), inflationary cosmology has been extremely successful from a phenomenological point of view. It predicted the spatial flatness of the universe, and - most importantly - provided the first theory for the origin of the large-scale structure of the universe based on fundamental physics [5, 6] (see also [2, 7] for qualitative arguments and [8] for an analysis of the predicted spectrum of gravitational waves based on the model of [3]). This theory predicted an almost scale-invariant spectrum of cosmological fluctuations, a prediction which has now been spectacularly confirmed in recent observations [9, 10].

Inflationary cosmology is based on the idea that there was a period in the very early universe during which space expanded almost exponentially. How to obtain a period of inflation based on fundamental physics has, however, been a more difficult question to address. In order to obtain a period of accelerated expansion in the context of Einstein's theory of General Relativity, the dominant component of matter needs to have an equation of state with sufficiently negative pressure p. More specifically, the inequality $p < -1/3\rho$ (where ρ is the energy density) is required. The standard approach to obtain inflation is to invoke the existence of a new scalar field φ, the *inflaton*, a scalar field which has to be very weakly coupled to the standard model particle physics sector. If the energy density of the scalar field is dominated by its potential energy, then a period of inflation will result.

At this point, the difficulty is shifted to the problem of obtaining a situation where the energy of a scalar field is dominated by the potential energy term for a sufficiently long period. The initial scenario of inflation [1, 2], in which the scalar field was assumed to be trapped in a false vacuum, did not work

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since the period of inflation terminated by bubble nucleation, and each post-inflationary bubble was too small to produce the present universe (see e.g. [11, 12, 14] for reviews of inflationary cosmology). The next models of inflation ("new inflation" [15, 16]) required initial conditions for the fields which had to be carefully tuned [17, 18]. This led to the development of inflationary models like chaotic inflation [19] and hybrid inflation [20], where inflation happens due to the slow rolling of a field ϕ from initially large values of $|\phi|$ (larger than the Planck scale in the case of chaotic inflation, smaller in the case of hybrid inflation, a scenario which invokes the existence of more than one scalar field). These latter models are much less sensitive to initial conditions, as shown in [21, 22].

As will be argued in the following section, whereas scalar field-driven inflationary cosmology (in the context of four space-time dimensional physics using no tools beyond ordinary quantum field theory) is a very successful scenario, it suffers from serious conceptual problems and thus cannot provide the final theory of the early universe. Since several of the key conceptual problems discussed below relate to ultraviolet issues, it is likely that the same new fundamental physics required to address the ultraviolet problems of the Standard Model (SM) of particle physics will be needed to develop a true "theory" of the very early universe. String theory is currently our best candidate for resolving the ultraviolet problems of the particle physics SM, and thus string theory may also provide us with the theory of the primordial universe, a theory which may well lead to a period of cosmological inflation - although the possible existence of alternative scenarios should not be discarded.

In this talk, I first discuss some of the conceptual problems of conventional inflationary cosmology. Then, I formulate some key challenges for any approach to string cosmology. "String Gas Cosmology" (SGC), a particular approach to combining string theory and cosmology first put forwards in [23], will be briefly reviewed in Section 4, and I will conclude with a summary of recent progress in and current problems of SGC. This article is the first of three reviews. The second [24] will focus on the principles of SGC, while the third [25] will concentrate on recent progress on the issue of moduli stabilization in SGC.

2. Conceptual Problems of Inflationary Cosmology

Before discussing some key conceptual problems of conventional scalar field-driven inflationary cosmology, let us recall some of the key features of cosmological inflation. To set our notation, we use the following metric for the homogeneous and isotropic background space-time:

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 , \tag{1}$$

where $d\mathbf{x}^2$ is the metric of $\mathcal{R}^3$ (we assume for simplicity a spatially flat universe), and $a(t)$ is the scale factor of the universe,

Figure 1 is a sketch of the space-time structure of an inflationary universe. The vertical axis is time, the horizontal axis is physical length. The time period between t_i and t_R is the period of inflation (here for simplicity taken to be exponential). During the period of inflation, the Hubble radius $l_H(t) \equiv H^{-1}(t)$, where $H(t) \equiv \dot{a}(t)/a(t)$, is constant. After inflation, the Hubble radius increases linearly in time. In contrast, the physical length corresponding to a fixed co-moving scale increases exponentially during the period of inflation, and then grows either as $t^{1/2}$ (radiation-dominated phase) or $t^{2/3}$ (matter-dominated phase), i.e. less fast than the Hubble radius.

The key feature of inflationary cosmology which can be seen from Figure 1 is the fact that fixed comoving scales are red-shifted exponentially relative to the Hubble radius during the period of inflation. Provided that the period of inflation lasted more than about 50 Hubble expansion times (this number is obtained assuming that the energy scale of inflation is of the order of 10^{16}GeV), then modes with a current wavelength up to the Hubble radius started out at the beginning of the period of inflation with a wavelength smaller than the Hubble radius at that time. Thus, it is possible to imagine a microscopic mechanism for creating the density fluctuations in the early universe which evolve into the cosmological structures we observe today.
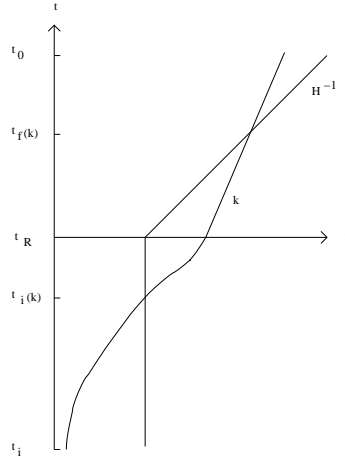
Fig. 1. Space-time diagram (sketch) showing the evolution of scales in inflationary cosmology. The vertical axis is time, and the period of inflation lasts between t_i and t_R, and is followed by the radiation-dominated phase of standard big bang cosmology. During exponential inflation, the Hubble radius H^{-1} is constant in physical spatial coordinates (the horizontal axis), whereas it increases linearly in time after t_R. The physical length corresponding to a fixed comoving length scale labelled by its wavenumber k increases exponentially during inflation but increases less fast than the Hubble radius (namely as $t^{1/2}$), after inflation.

Since during the period of inflation any pre-existing ordinary matter fluctuations are red-shifted, it is reasonable to assume that quantum vacuum fluctuations are the source of the currently observed structures [5, 6] (see also [7]). The time-translational symmetry of the inflationary phase leads, independent of a precise understanding of the generation mechanism for the fluctuations, to the prediction that the spectrum of cosmological perturbations should be approximately scale-invariant [7, 2].

The quantum theory of linearized cosmological perturbations [26, 27], in particular applied to inflationary cosmology, has in the mean time become a well-developed research area (see e.g. [28] for a detailed review, and [29] for a pedagogical introduction). For simple scalar field matter, there is a single canonically normalized variable, often denoted by v, which carries the information about the "scalar metric fluctuations", the part of the metric perturbations which couples at linearized level to the matter. The equation of motion for each Fourier mode of this variable v has the form of a harmonic oscillator with a time-dependent square mass m^2, whose form is set by the cosmological background. On scales smaller than the Hubble radius, the modes oscillate (quantum vacuum oscillations). However, on length scales larger than the Hubble radius, m^2 is negative, the oscillations cease, and the wave functions of these modes undergo squeezing. Since the squeezing angle in phase space does not depend on the wave number, all modes re-enter the Hubble radius at late times with the same squeezing angle. This then leads to the prediction of "acoustic" oscillations in the angular power spectrum of CMB anisotropies (see e.g. [30] for a recent analytical treatment), a prediction spectacularly confirmed by the WMAP data [10], and allowing cosmologists to fit for several important cosmological parameters.

In spite of this spectacular phenomenological success of the inflationary paradigm, I will now argue that conventional scalar field-driven inflation suffers from several important conceptual problems.

The first problem (the **amplitude problem**) relates to the amplitude of the spectrum of cosmological perturbations. In a wide class of inflationary models, obtaining the correct amplitude requires the introduction of a hierarchy in scales, namely [31]

$$\frac{V(\varphi)}{\Delta\varphi^4} \le 10^{-12}, \tag{2}$$

where $\Delta\varphi$ is the change in the inflaton field during one Hubble expansion time (during inflation), and $V(\varphi)$ is the potential energy during inflation.

A more serious problem is the **trans-Planckian problem** [32]. Returning to the space-time diagram of Figure 1, we can immediately deduce that, provided that the period of inflation lasted sufficiently long (for GUT scale inflation the number is about 70 e-foldings), then all scales inside of the Hubble radius today started out with a physical wavelength smaller than the Planck scale at the beginning of inflation. Now, the theory of cosmological perturbations is based on Einstein's theory of General Relativity coupled to a simple semi-classical description of matter. It is clear that these building blocks of the theory are inapplicable on scales comparable and smaller than the Planck scale. Thus, the key successful prediction of inflation (the theory of the origin of fluctuations) is based on suspect calculations since new physics *must* enter into a correct computation of the spectrum of cosmological perturbations. The key question is as to whether the predictions obtained using the current theory are sensitive to the specifics of the unknown theory which takes over on small scales.

One approach to study the sensitivity of the usual predictions of inflationary cosmology to the unknown physics on trans-Planckian scales is to study toy models of ultraviolet physics which allow explicit calculations. The first approach which was used [33, 34] is to replace the usual linear dispersion relation for the Fourier modes of the fluctuations by a modified dispersion relation, a dispersion relation which is linear for physical wavenumbers smaller than the scale of new physics, but deviates on larger scales. Such dispersion relations were used previously to test the sensitivity of black hole radiation on the unknown physics of the UV [35, 36]. It was found [33] that if the evolution of modes on the trans-Planckian scales is non-adiabatic, then substantial deviations of the spectrum of fluctuations from the usual results are possible. Non-adiabatic evolution turns an initial state minimizing the energy density into a state which is excited once the wavelength becomes larger than the cutoff scale. Back-reaction effects of these excitations may limit the magnitude of the trans-Planckian effects, but - based on our recent study [37] - not to the extent initially assumed [38, 39]. Other approaches to study the trans-Planckian problem have been pursued, e.g. based on implementing the space-space [40] or space-time [41] uncertainty relations, on a minimal length hypothesis [42], on "minimal trans-Planckian" assumptions (taking as initial conditions some vacuum state at the mode-dependent time when the wavelength of the mode is equal to the Planck scale [43], or on effective field theory [44], all showing the possibility of trans-Planckian corrections.

A third problem is the **singularity problem**. It was known for a long time that standard Big Bang cosmology cannot be the complete story of the early universe because of the initial singularity, a singularity which is unavoidable when basing cosmology on Einstein's field equations in the presence of a matter source obeying the weak energy conditions (see e.g. [45] for a textbook discussion). Recently, the singularity theorems have been generalized to apply to Einstein gravity coupled to scalar field matter, i.e. to scalar field-driven inflationary cosmology [46]. It is shown that in this context, a past singularity at some point in space is unavoidable. Thus we know, from the outset, that scalar field-driven inflation cannot be the ultimate theory of the very early universe.

The Achilles heel of scalar field-driven inflationary cosmology is, however, the **cosmological constant problem**. We know from observations that the large quantum vacuum energy of field theories does not gravitate today. However, to obtain a period of inflation one is using the part of the energy-momentum tensor of the scalar field which looks like the vacuum energy. In the absence of a convincing solution of the cosmological constant problem it is unclear whether scalar field-driven inflation is robust, i.e. whether the mechanism which renders the quantum vacuum energy gravitationally inert today will not also prevent the vacuum energy from gravitating during the period of slow-rolling of the inflaton field. Note that the approach to addressing the cosmological constant problem making use of the gravitational back-reaction of long range fluctuations (see [47] for a summary of this approach) does not prevent a long period of inflation in the early universe.

Finally, a key challenge for conventional inflationary cosmology is to find a well-motivated candidate for the scalar field which drives inflation, the inflaton. Ever since the failure of the model of *old inflation* [1, 2], it is clear that physics beyond the Standard Model of particle physics must be invoked.

3. Challenges for String Cosmology

In the following we will focus not on the question of how to obtain inflation from a new fundamental theory of micro-physics, but rather on how such new micro-physics can help resolve some of the key conceptual problems of scalar field-driven inflation listed in the previous section. To be specific, we will assume that the new fundamental physics is based on superstring theory.

Why could superstring theory help resolve the key problems mentioned in the previous section? First, an effective field theory derived from string theory contains many scalar fields which are massless before supersymmetry breaking. Thus, the hierarchy of scales required to produce the observed small amplitude of cosmological perturbations might arise naturally. Second, string theory is supposed to provide a theory which describes physics on all scales. In such a theory, it would be possible to track the cosmological perturbations for all times. We would know the trans-Planckian physics and would thus be able to compute the trans-Planckian signatures. Thirdly, one of the goals of string theory is to resolve all singularities. If this goal can be achieved (and a scenario which can eliminate cosmological singularities is discussed below), then it would be possible to construct a theory of the early universe which does not suffer from the singularity problem of scalar field-driven inflation. At the moment, it is not clear whether string theory can address the cosmological constant problem. It is possible, however, that the cosmological constant problem can be cured by a better understanding of the infrared physics in the context of our existing micro-physical theory (see [47] for a review and [48] for a key original work).

An immediate problem which arises when trying to connect string theory with cosmology is the **dimensionality problem**. Superstring theory is perturbatively consistent only in 10 space-time dimensions, but we only see 3 large spatial dimensions. The original approach to addressing this problem is to assume that the 6 extra dimensions are compactified on a very small space which cannot be probed with our available energies. However, from the point of view of cosmology, it is quite unsatisfactory not to be able to understand why it is precisely 3 dimensions which are not compactified and why the compact dimensions are stable. Brane world cosmology [49] provides another approach to this problem: it assumes that we live on a three-dimensional brane embedded in a large nine-dimensional space. Once again, a cosmologically satisfactory theory should explain why it is likely that we will end up exactly on a three-dimensional brane (for some interesting work addressing this issue see [50, 51, 52]).

Finding a natural solution to the dimensionality problem is thus one of the key challenges for superstring cosmology. This challenge has various aspects. First, there must be a mechanism which singles out three dimensions as the number of spatial dimensions we live in. Second, the moduli fields which describe the volume and the shape of the unobserved dimensions must be stabilized (any strong time-dependence of these fields would lead to serious phenomenological constraints). This is the **moduli problem** for superstring cosmology. As mentioned above, resolving the **singularity problem** is another of the main challenges. These are the three problems which **string gas cosmology** [23, 53, 54] explicitly addresses at the present level of development..

In order to make successful connection with late time cosmology, any approach to string cosmology must also solve the **flatness problem**, namely make sure that the three large spatial dimensions obtain a sufficiently high entropy (size) to explain the current universe. Finally, it must provide a mechanism to produce a nearly scale-invariant spectrum of nearly adiabatic cosmological perturbations.

Since superstring theory leads to many light scalar fields, it is possible that superstring cosmology will provide a convincing realization of inflation (see e.g. [55] for reviews of recent work attempting to obtain inflation in the context of string theory). However, it is also possible that superstring cosmology will provide an alternative to cosmological inflation, maybe along the lines of the Pre-Big-Bang [56] or

Ekpyrotic [57] scenarios. The greatest challenge for these alternatives is to solve the flatness problem (see e.g. [58]).

4. Overview of String Gas Cosmology

A key obstacle towards making progress in developing superstring cosmology is that non-perturbative string theory does not yet exist. It is likely that the new structures of non-perturbative string theory will reveal quite new and unexpected possibilities for cosmology in a similar way that the non-perturbative understanding of quantum particle dynamics was crucial in developing inflationary cosmology.

In the absence of a non-perturbative formulation of string theory, the approach to string cosmology which we have suggested [23, 53, 54] (see also [59]) is to focus on symmetries and degrees of freedom which are new to string theory (compared to point particle theories) and which will be part of a non-perturbative string theory, and to use them to develop a new cosmology. The symmetry we make use of is **T-duality**, and the new degrees of freedom are **string winding modes**.

Let us assume that all spatial directions are toroidal, with R denoting the radius of the torus. Strings have three types of states: *momentum modes* which represent the center of mass motion of the string, *oscillatory modes* which represent the fluctuations of the strings, and *winding modes* counting the number of times a string wraps the torus. Both oscillatory and winding states are special to strings as opposed to point particles.

The energy of an oscillatory mode is independent of R, momentum mode energies are quantized in units of $1/R$, i.e.

$$E_n = n\frac{1}{R}, \tag{3}$$

and winding mode energies are quantized in units of R:

$$E_m = mR, \tag{4}$$

where both n and m are integers.

The T-duality symmetry is a symmetry of the spectrum of string states under the change

$$R \rightarrow \frac{1}{R} \tag{5}$$

in the radius of the torus (in units of the string length l_s). Under such a change, the energy spectrum of string states is invariant: together with the transformation (5), winding and momentum quantum numbers need to be interchanged

$$(n, m) \rightarrow (m, n). \tag{6}$$

The string vertex operators are consistent with this symmetry, and thus T-duality is a symmetry of perturbative string theory. Postulating that T-duality extends to non-perturbative string theory leads [60] to the need of adding D-branes to the list of fundamental objects in string theory. With this addition, T-duality is expected to be a symmetry of non-perturbative string theory.

One deficiency inherent in any approach to superstring cosmology in the absence of a non-perturbative formulation of string theory is the requirement to introduce a classical background. We choose the background to be dilaton gravity. It is crucial to include the dilaton in the Lagrangian, firstly since the dilaton arises in string perturbation theory at the same level as the graviton, and secondly because it is only the action of dilaton gravity (rather than the action of Einstein gravity) which is consistent with the T-duality symmetry. Given this background, we consider an ideal gas of matter made up of all fundamental states of string theory, in particular including string winding modes.

Any physical theory requires initial conditions. We assume that the universe starts out small and hot. For simplicity, we take space to be toroidal, with radii in all spatial directions given by the string

scale. We assume that the initial energy density is very high, with an effective temperature which is close to the Hagedorn temperature [61], the maximal temperature of perturbative string theory. It is not important for us to have perfect thermal equilibrium, but we need to assume that all string states (including states with winding and momentum) are initially excited.

The first predictions of string gas cosmology (SGC) were reached by heuristic considerations [23]. Based on the T-duality symmetry, it was argued that the cosmology resulting from SGC will be non-singular. For example, as the background radius R varies, the physical temperature T will obey the symmetry

$$T(R) = T(1/R) \tag{7}$$

and thus remain non-singular even if R decreases to zero. Similarly, the length L measured by a physical observer will be consistent with the symmetry (5), hence realizing the idea of a minimal physical length.

Next, it was argued [23] that in order for spatial sections to become large, the winding modes need to decay. This decay, at least on a background with stable one cycles such as a torus, is only possible if two winding modes meet and annihilate. Since string world sheets have measure zero probability for intersecting in more than four space-time dimensions, winding modes can annihilate only in three spatial dimensions (see, however, the recent caveats to this conclusion based on the work of [62]). Thus, only three spatial dimensions can become large, hence explaining the observed dimensionality of space-time. As was shown later [54], adding branes to the system does not change these conclusions since at later times the strings dominate the cosmological dynamics. Note that in the three dimensions which are becoming large there is a natural mechanism of isotropization as long as some winding modes persist [63].

The equations of SGC are based on coupling an ideal gas of all string and brane modes to the background space-time of dilaton gravity. These equations were first studied in [53] (see also [64]). In the context of a homogeneous and isotropic background metric, it follows, as will be reviewed in [24], that - in the absence of string interactions - the scale factor remains bounded from above and below. String winding modes resist the expansion, string momentum modes resist the contraction. Given a gas of strings with equal numbers of winding and momentum modes, the equilibrium state for the radius is the string scale (the "self-dual" radius). Note that the dilaton is not stabilized at this stage.

The interaction of string winding modes results in string loop production. This process was studied in the three large dimensions in [65], with the conclusion that, after a stage of "loitering", the three spatial dimensions are liberated and expand, as long as the dilaton is not too small (this last caveat follows from the work of [62]).

5. Progress and Problems in String Gas Cosmology

A key issue in all approaches to string cosmology is the question of **moduli stabilization**. The challenge is to fix the shape and volume moduli of the compact dimensions and to fix the value of the dilaton. Moduli stabilization is essential to obtain a consistent late time cosmology.

There has recently been a lot of progress on the issue of moduli stabilization in SGC, progress which will be reviewed in detail in [25]. In a first study [66], the stabilization of the radii of the extra dimensions (the "radion" degrees of freedom) was studied in the string frame. It was shown that, as long as there are an equal number of string momentum and winding modes about the compact directions, the radii are dynamically stabilized at the self-dual radius. The dilaton, however, is in general evolving in time.

For late time cosmology, it is crucial to show that the radion degrees of freedom are stabilized in the Einstein frame. Obstacles towards achieving this goal were put forward in [67, 68, 69, 70]. However, if the spectrum of string states contains modes which are massless at the self-dual radius (which is the case for heterotic but not for Type II string theory), then these modes generate an effective

potential for the radion which has a minimum with vanishing energy at the self-dual radius and thus yield radion stabilization [71, 72] (see also [73]). As shown in these references, the radion stabilization mechanism is consistent with late time cosmology (e.g. fifth force constraints). These same massless modes also yield stabilization of the shape moduli [74] (see also [75] for a study which appeared after the conference). The outstanding challenge in this approach is to stabilize the dilaton (for some ideas see [76]).

There has been a substantial amount of recent work on SGC. For lack of space, references to this work will be given in [24]. Although the moduli stabilization mechanism in SGC is conceptually simpler than the mechanisms used in other approaches to string cosmology (see e.g. [77]) in that it does not use fluxes or warping of the internal dimensions, that is uses more string-specific ingredients, and that it is based on a full dynamical analysis, the applicability of the mechanism at first sight depends on special topological features of the internal space, namely on the existence of stable one cycles. However, as shown in [78], the basic mechanisms of SGC generalize to certain orbifolds. They may also be generalized using ideas of [79] to more general Calabi-Yau spaces.

A crucial challenge for SGC is to find a solution to the flatness or entropy problem, namely to produce a sufficiently large three-dimensional universe to explain its observed size. The only avenue to achieve this which is well-established at the present time is by invoking a period of inflation of the three large dimensions at later times. For some ideas on how to obtain inflation from SGC see [80, 81, 82]. A challenge for inflation in the context of SGC is to ensure that inflation is compatible with radion stabilization. Note, however, that since SGC naturally seems to point to a bouncing cosmology, eventually an alternative to inflation in the context of SGC may emerge. In this case, however, the challenge would be to find an alternative mechanism for generating a nearly scale-invariant spectrum of nearly adiabatic cosmological fluctuations.

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